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# Fermion tunneling from anti-de Sitter spaces

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**Abstract.** Kerner and Mann's recent research on the Hawking radiation of the spherically symmetric uncharged black hole shows that the Hawking temperature can be obtained by the fermion tunneling method. In this paper, we extend this work to the general case and view the Hawking radiations of the Reissner–Nordström black hole, Kerr black hole and Kerr–Newman black hole in anti-de Sitter spaces. The Hawking temperatures are recovered and are exactly the same as that obtained by other methods.

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## 1 Introduction

The thermodynamic properties of black holes are an important subject in black hole physics. In the past several decades, people have done much research on thermal radiation of black holes [1–9]. However, the tunneling method has not strictly been adopted in their treatments despite that the Hawking radiation is described as a quantum tunneling process triggered by vacuum fluctuations near the horizon. Recently, a method to describe Hawking radiation as a tunneling process, where a particle moves in dynamical geometry, has been developed [10–14]. Applying this method, Parikh and Wilczek studied the Hawking radiation of spherically symmetric black holes [12–14]. The result shows that when the unfixed background space-time and self-gravitational interaction are taken into account, the actual Hawking radiation spectrum deviates from the purely thermal spectrum, which satisfies the underlying unitary theory and gives a leading correction to the radiation spectrum. Moreover, they pointed out that the tunneling potential barrier is afforded by the outgoing particle itself, and thus the producing mechanism of the potential barrier was resolved. Extending this method to the general case, people investigated the Hawking radiation of various space-times [15–22]. At the same time, the anomaly method [23] to derive the Hawking radiation attracted attention [24]. In this work, the Hawking radiation is understood as a compensating flux to cancel anomalies at the horizon. To avoid breakdown of general covariance and gauge invariance at the quantum level, the total flux of Hawking radiation in each outgoing partial wave of a charged quantum field in the charged black hole background must be equal to that of a $1+1$ dimensional blackbody at the Hawking temperature with the appropriate chemical potential. For the rotating case, the partial wave of the quantum fields in the rotating space-time background can be interpreted as a $1+1$ dimensional charged field with a charge proportional to the azimuthal angular momentum $m$. Applying the anomaly method, people had much success and all of this has contributed to understanding of the thermodynamic properties of black holes [25–31].

Black holes radiate not only scalar particles but also Dirac particles. Recently, Kerner and Mann have studied the Hawking radiation of spin 1/2 particles of the spherically symmetric uncharged black hole and recovered the Hawking temperature of the black hole by fermion tunneling [32]. In their treatment, they assumed that the number of outgoing particles with spin up is statistically equal to that of the spin down case and they assumed that the angular momentum does not change. Moreover, the back reaction and self-gravitational interaction were neglected; thus, the radiation spectrum is only the leading term. This is the first paper adopting the tunneling method to study the Hawking radiation of fermions. However, the cases of charged and rotating black holes have not been investigated and the difficulties consist in how to consider the effect of the electromagnetic field and the dragging effect.

Our work in this paper is to extend Kerner and Mann's work to the charged and rotating anti-de Sitter spaces and view the Hawking radiation of the Reissner–Nordström anti-de Sitter black hole and Kerr anti-de Sitter black hole as well as the Kerr–Newman anti-de Sitter black hole by the fermion tunneling method. As a result, the Hawking temperatures are recovered and are fully in consistence with that obtained by other methods. The solutions of black holes in anti-de Sitter spaces come from the Einstein equations with a negative cosmological constant. Anti-de

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Sitter black holes are different from de Sitter black holes, the difference consisting in them having minimum temperatures occurring when their sizes are of the order of the characteristic radius of the anti-de Sitter space. For larger anti-de Sitter black holes, their red-shifted temperatures measured at infinity are greater. This implies that such black holes can be in stable equilibrium with thermal radiation at a certain temperature. Moreover, recent developments in string/M-theory greatly stimulate the study of black holes in anti-de Sitter spaces. One example is the AdS/CFT correspondence between a weakly coupled gravity system in an anti-de Sitter (AdS) background and a strongly coupled conformal field theory (CFT) on its boundary. Therefore the study of anti-de Sitter black holes is necessary and meaningful.

The remainder of this paper can be outlined as follows. In the next section, taking the electromagnetic field effect into account, we view the Hawking radiation of the spherically symmetric charged black hole in anti-de Sitter space and recover the Hawking temperature of the black hole by the fermion tunneling method. In Sects. 3 and 4, the Hawking radiation of fermions for the Kerr and Kerr–Newman black holes in anti-de Sitter spaces are investigated, respectively. After performing the dragging coordinate transformation, the Kerr anti-de Sitter black hole and Kerr–Newman anti-de Sitter black hole have a similar form of metric. Thus we choose similar $\gamma^\mu$ matrices in the treatment. This is very helpful to investigate the Hawking radiation. Section 5 contains some discussion and our conclusion.

## 2 Reissner–Nordström anti-de Sitter black hole

The Hawking radiation of scalar particles for the Reissner–Nordström black hole in anti-de Sitter space has been discussed. In this section, we focus our attention on the case of fermions. The metric of the Reissner–Nordström anti-de Sitter black hole is given by [33]

$$\begin{aligned} \mathrm{d}s^2 &= -f(r)\,\mathrm{d}t^2 + f^{-1}(r)\,\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\,,\\ f(r) &= 1 - \frac{r_+}{r} - \frac{r_+^3}{R^2 r} - \frac{Q^2}{r_+ r} + \frac{Q^2}{r^2} + \frac{r^2}{R^2}\,, \end{aligned} \tag{1}$$

where $r_+$, $Q$ and $A_\mu = A_t\,\mathrm{d}t = \frac{Q}{r}\,\mathrm{d}t$ are the event horizon, the electric charge and the electric potential of the black hole, respectively; $R$ is the radius, which is related to the cosmological constant by $R^2 = -\frac{3}{\Lambda}$. The physical mass of the black hole is

$$M = \frac{1}{2}\left(r_+ + \frac{r_+^3}{R^2} + \frac{Q^2}{r_+}\right).$$

There are some special properties in the metric (1): the event horizon is coincident with the infinite red-shift surface, the metrics satisfy Landau's condition of coordinate clock synchronization, and so on. These properties are helpful to view the Hawking radiation of the black hole.

The Dirac equation of a charged particle in an electromagnetic field can be written as

$$\mathrm{i}\gamma^\mu\left(\partial_\mu + \Omega_\mu + \frac{\mathrm{i}}{\hbar}eA_\mu\right)\psi + \frac{m}{\hbar}\psi = 0\,, \tag{2}$$

where $\Omega_\mu = \frac{\mathrm{i}}{2}\Gamma_\mu^{\alpha\beta}\sum_{\alpha\beta}$, $\sum_{\alpha\beta} = \frac{\mathrm{i}}{4}[\gamma^\alpha, \gamma^\beta]$, $\gamma^\mu$ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, and $e$ and $A_\mu$ are the charge of outgoing particle and electromagnetic potential of the black hole, respectively. To resolve the Dirac equation, we should choose appropriate $\gamma^\mu$ matrices and our choice is given as

$$\begin{aligned} \gamma^t &= \frac{1}{\sqrt{f(r)}}\begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}, \quad \gamma^\theta = \frac{1}{r}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix},\\ \gamma^r &= \sqrt{f(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^\varphi = \frac{1}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \end{aligned} \tag{3}$$

and $\sigma^\mu$ is the Pauli sigma matrix. $\gamma^5 = \mathrm{i}\gamma^t\gamma^r\gamma^\theta\gamma^\varphi$ is the resulting $\gamma^5$ matrix. For the spin 1/2 particle, there are two states, corresponding to spin up (↑) and spin down (↓), and two wave functions correspond to this,

$$\psi_{(\uparrow)} = \begin{pmatrix} A \\ 0 \\ B \\ 0 \end{pmatrix} \exp\left(\frac{\mathrm{i}}{\hbar}I_\uparrow\right), \quad \psi_{(\downarrow)} = \begin{pmatrix} 0 \\ C \\ 0 \\ D \end{pmatrix} \exp\left(\frac{\mathrm{i}}{\hbar}I_\downarrow\right), \tag{4}$$

in which $I$, $\psi$, $A$, $B$, $C$ and $D$ are functions of $t$, $r$, $\theta$ and $\varphi$. In the following, the key is to explore the action. According to [32], we assume that the angular momentum of the black hole does not change in this paper. We first explore the action of a spin up particle. Inserting the wave function and the $\gamma^\mu$ matrices into the Dirac equation yields

$$-\left(\frac{\mathrm{i}A}{\sqrt{f(r)}}(\partial_t I_\uparrow + eA_t) + B\sqrt{f(r)}\partial_r I_\uparrow\right) + mA = 0\,, \tag{5}$$

$$\left(\frac{\mathrm{i}B}{\sqrt{f(r)}}(\partial_t I_\uparrow + eA_t) - A\sqrt{f(r)}\partial_r I_\uparrow\right) + mB = 0\,, \tag{6}$$

$$-B\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{\mathrm{i}}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0\,, \tag{7}$$

$$-A\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{\mathrm{i}}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0\,. \tag{8}$$

From the above equations, we can find that it is difficult to solve the action directly, but separation of variables can be carried out. Moreover, there are four equations, but our interest is in the first two. Considering the properties of the Reissner–Nordström black hole in anti-de Sitter space, we carry out the separation of variables as follows:

$$I_\uparrow = -\omega t + W(r) + \Theta(\theta, \varphi)\,, \tag{9}$$

where $\omega$ is the energy of the particle. Substituting (9) into (5) and (6), we have

$$\left(\frac{\mathrm{i}A}{\sqrt{f(r)}}(\omega-eA_t)-B\sqrt{f(r)}\partial_r W(r)\right)+mA=0\,, \tag{10}$$

$$-\left(\frac{\mathrm{i}B}{\sqrt{f(r)}}(\omega-eA_t)+A\sqrt{f(r)}\partial_r W(r)\right)+mB=0\,. \tag{11}$$

When $m=0$, this is the Hawking radiation of the massless particles, and then the electric charge $e$ and electric potential in corresponding equations should be zero and can be deleted. When $m\neq 0$, it is the Hawking radiation of a charged particle. In the case of $m\neq 0$, solving $W(r)$ yields

$$W(r)=\pm\int\frac{\sqrt{(\omega-eA_t)^2+m^2 f(r)}}{f(r)}\mathrm{d}r \\ =\pm\mathrm{i}\pi\frac{\omega-eA_+}{\frac{1}{r_+}+\frac{3r_+}{R^2}-\frac{Q^2}{r_+^3}}\,, \tag{12}$$

where $+$ $(-)$ denotes the outgoing (ingoing) solution, and $A_+$ is the electric potential at the event horizon. Substituting (12) into (9), we can get the imaginary part of the action as

$$\operatorname{Im} I_\pm=\pm\pi\frac{\omega-eA_+}{\frac{1}{r_+}+\frac{3r_+}{R^2}-\frac{Q^2}{r_+^3}}+\operatorname{Im}K\,, \tag{13}$$

where $\operatorname{Im}K$ is the contribution of $\Theta(\theta,\varphi)$. Thus, the tunneling probability of the fermion is

$$\Gamma=\frac{P(\text{emission})}{P(\text{absorption})}=\frac{\exp(-2\operatorname{Im}I_+)}{\exp(-2\operatorname{Im}I_-)} \\ =\exp\left(-4\pi\frac{\omega-eA_+}{\frac{1}{r_+}+\frac{3r_+}{R^2}-\frac{Q^2}{r_+^3}}\right). \tag{14}$$

So the Hawking temperature is recovered:

$$T=\frac{1+\frac{3r_+^2}{R^2}-\frac{Q^2}{r_+^2}}{4\pi r_+}\,, \tag{15}$$

which is full in accordance with that obtained by other methods, and the kind of emission particles does not matter. From (15), we can find that the minimum value of the Hawking temperature $T_0$ is located at $r^2=r_+^2=\frac{R^2}{6}+\left(\frac{R^4}{36}-Q^2R^2\right)^{\frac{1}{2}}$. For the spin down case, we can adopt the same process and get the same result. In this paper, we will not investigate it in detail.

## 3 Kerr anti-de Sitter black hole

In this section, we focus our attention on the Hawking radiation of the uncharged rotating black hole in the anti-de Sitter space. The metric of the Kerr anti-de Sitter black hole was given by Carter [34], which is

$$\mathrm{d}s^2=-\frac{\Delta}{\rho^2}\left(\mathrm{d}t-\frac{a}{\Xi}\sin^2\theta\,\mathrm{d}\varphi\right)^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2+\frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 \\ +\frac{\Delta_\theta\sin^2\theta}{\rho^2}\left(a\,\mathrm{d}t-\frac{r^2+a^2}{\Xi}\mathrm{d}\varphi\right)^2, \tag{16}$$

where

$$\Delta=(r^2+a^2)(1+r^2l^{-2})-2Mr\,,\qquad \Xi=1-a^2l^{-2}\,, \\ \rho^2=r^2+a^2\cos^2\theta\,,\qquad \Delta_\theta=1-a^2l^{-2}\cos^2\theta\,, \tag{17}$$

$a$ is angular momentum parameter and $l$ is a constant related to the cosmological factor as $\Lambda=-3l^{-2}$. The angular momentum parameter must satisfy the relation $a^2<l^2$. When they approach the critical limit $a^2=l^2$, the metric becomes singular. The event horizon is located at $r=r_+$. Since the black hole rotates, there exists a frame dragging effect of the coordinate system in the rotating space-time, and the matter field in the ergosphere near the horizon must be dragged by the gravitational field with an azimuthal angular velocity, which is not convenient for us discussing Hawking radiation. Therefore, we perform a dragging coordinate transformation:

$$\phi=\varphi-\Omega t,\quad \Omega=\frac{\left[\Delta_\theta(r^2+a^2)-\Delta\right]a\Xi}{\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta}\,. \tag{18}$$

Then the metric takes the form

$$\mathrm{d}s^2=-F(r)\,\mathrm{d}t^2+\frac{1}{G(r)}\mathrm{d}r^2+\frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 \\ +\frac{\sin^2\theta}{\rho^2\Xi^2}\left[\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta\right]\mathrm{d}\phi^2\,, \tag{19}$$

where

$$F(r)=\frac{\Delta\Delta_\theta\rho^2}{\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta},\quad G(r)=\frac{\Delta}{\rho^2}\,.$$

Comparing the metric (19) with (1), we find that they have similar forms of the metric. So they have similar properties, namely the event horizon coincides with the infinite red-shift surface; the metrics satisfy Landau's condition of the coordinate clock synchronization. This is convenient in the choice of the $\gamma^\mu$ matrices and investigating Hawking radiation.

The equation of motion of the spin 1/2 particle satisfies

$$\mathrm{i}\gamma^\mu(\partial_\mu+\Omega_\mu)\psi+\frac{m}{\hbar}\psi=0\,. \tag{20}$$

Considering a similar form as the metrics of (19) and (1), we can choose similar $\gamma^\mu$ matrices here as

$$\gamma^t = \frac{1}{\sqrt{F(r)}}\begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}, \quad \gamma^r = \sqrt{G(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix},$$
$$\gamma^\theta = \sqrt{\frac{\Delta_\theta}{\rho^2}}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix},$$
$$\gamma^\phi = \sqrt{\frac{\rho^2 \Xi^2}{\sin^2\theta\left[\Delta_\theta (r^2+a^2)^2 - \Delta a^2 \sin^2\theta\right]}}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}. \tag{21}$$

The wave functions are also given by (4). Substituting the spin up wave function and the $\gamma^\mu$ matrices into the Dirac equation, we have

$$-\left(\frac{\mathrm{i}A}{\sqrt{F(r)}}\partial_t I_\uparrow + B\sqrt{G(r)}\partial_r I_\uparrow\right) + mA = 0, \tag{22}$$
$$\left(\frac{\mathrm{i}B}{\sqrt{F(r)}}\partial_t I_\uparrow - A\sqrt{G(r)}\partial_r I_\uparrow\right) + mB = 0, \tag{23}$$
$$-B\left(\sqrt{\frac{\Delta_\theta}{\rho^2}}\partial_\theta I_\uparrow + \mathrm{i}\sqrt{\frac{\rho^2\Xi^2}{\sin^2\theta\left[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]}}\partial_\phi I_\uparrow\right) = 0, \tag{24}$$
$$-A\left(\sqrt{\frac{\Delta_\theta}{\rho^2}}\partial_\theta I_\uparrow + \mathrm{i}\sqrt{\frac{\rho^2\Xi^2}{\sin^2\theta\left[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]}}\partial_\phi I_\uparrow\right) = 0. \tag{25}$$

Our interest is also in the inclusion of $\partial_r I_\uparrow$, namely (22) and (23). Considering the properties of the Kerr anti-de Sitter space-time, we carry out a separation of the variables as follows:

$$I_\uparrow = -(\omega - j\Omega)t + W(r) + j\phi + \Theta(\theta), \tag{26}$$

where $\omega$ and $j$ are the energy and magnetic quantum number of the radiation particle, respectively. Inserting (26) into (22) and (23), we have

$$\left(\frac{\mathrm{i}A}{\sqrt{F(r)}}(\omega - j\Omega) - B\sqrt{G(r)}\partial_r W(r)\right) + mA = 0, \tag{27}$$
$$\left(-\frac{\mathrm{i}B}{\sqrt{F(r)}}(\omega - j\Omega) - A\sqrt{G(r)}\partial_r W(r)\right) + mB = 0. \tag{28}$$

When $m = 0$, (27) and (28) decouple and we have the Hawking radiation of a massless particle. When $m \neq 0$, they couple and we have Hawking radiation of the massive case. Solving $W(r)$ yields

$$W_\pm(r) = \pm\int\sqrt{\frac{(\omega - j\Omega)^2 + m^2F(r)}{F(r)G(r)}}\,\mathrm{d}r$$
$$= \pm\mathrm{i}\pi\frac{\omega - j\Omega_+}{\sqrt{F'(r_+)G'(r_+)}}$$
$$= \pm\mathrm{i}\pi\frac{(r_+^2+a^2)(\omega - j\Omega_+)}{r_+ + r_+(3r_+^2+a^2)l^{-2} - a^2r_+^{-1}}, \tag{29}$$

where $+/-$ are the outgoing/ingoing solutions, and $\Omega_+ = \frac{a\Xi}{r_+^2+a^2}$ is the angular velocity at the event horizon. Thus the tunneling probability of the fermion is

$$\Gamma = \frac{P(\text{emission})}{P(\text{absorption})} = \frac{\exp(-2\,\mathrm{Im}\,I_+)}{\exp(-2\,\mathrm{Im}\,I_-)}$$
$$= \exp\left(-4\pi\frac{(r_+^2+a^2)(\omega - j\Omega_+)}{r_+ + r_+(3r_+^2+a^2)l^{-2} - a^2r_+^{-1}}\right). \tag{30}$$

Then the Hawking temperature is gotten as

$$T = \frac{r_+ + r_+(3r_+^2+a^2)l^{-2} - a^2r_+^{-1}}{4\pi(r_+^2+a^2)}, \tag{31}$$

which is in consistence with that obtained by other methods [35]. For Hawking radiation of the spin down case the same process can be followed and the same result can be gotten.

## 4 Kerr–Newman anti-de Sitter black hole

In this section, we focus our attention on the charged rotating anti-de Sitter space-time and view the Hawking radiation of the Kerr–Newman anti-de Sitter black hole [34, 36]. The metric of the Kerr–Newman anti-de Sitter black hole can be given by replacing $\Delta$ with $\Delta = (r^2+a^2)(1+r^2l^{-2}) - 2Mr + Q^2$ in the metric (16), and its electromagnetic potential is

$$A_\mu = A_t\,\mathrm{d}t + A_\varphi\,\mathrm{d}\varphi = \frac{Qr}{\rho^2}\,\mathrm{d}t - \frac{Qra\sin^2\theta}{\rho^2\Xi}\,\mathrm{d}\varphi. \tag{32}$$

The event horizon is located at $\Delta = 0$. Since the Kerr and Kerr–Newman black hole in anti-de Sitter spaces have a similar form of metric and both of them express the rotating space-time, we perform a similar treatment. After introducing the dragging coordinate transformation $\phi = \varphi - \Omega t$, we get the metric form similar to the one given by (19). Note that $\Delta$ in $\Omega$, $F(r)$, $G(r)$ and in this section is $\Delta = (r^2+a^2)(1+r^2l^{-2}) - 2Mr + Q^2$. Correspondingly, the electromagnetic potential takes the form of

$$A_\mu = A_t\,\mathrm{d}t + A_\phi\,\mathrm{d}\phi$$
$$= \frac{Qr\Delta_\theta(r^2+a^2)}{\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta}\,\mathrm{d}t - \frac{Qra\sin^2\theta}{\rho^2\Xi}\,\mathrm{d}\phi. \tag{33}$$

To view the Hawking radiation of the charged fermion, we choose the Dirac equation given in (2). Considering the properties of the Kerr and Kerr–Newman anti-de Sitter black holes, we still choose similar $\gamma^\mu$ matrices and the wave function given in Sects. 2 and 3. Substituting them into the Dirac equation, we have

$$-\left(\frac{\mathrm{i}A}{\sqrt{F(r)}}(\partial_t I_\uparrow + eA_t) + B\sqrt{G(r)}\partial_r I_\uparrow\right) + mA = 0\,, \tag{34}$$

$$\left(\frac{\mathrm{i}B}{\sqrt{F(r)}}(\partial_t I_\uparrow + eA_t) - A\sqrt{G(r)}\partial_r I_\uparrow\right) + mB = 0\,, \tag{35}$$

$$-B\left(\sqrt{\frac{\Delta_\theta}{\rho^2}}\partial_\theta I_\uparrow + \mathrm{i}\sqrt{\frac{\rho^2\Xi^2}{\sin^2\theta\left[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]}}(\partial_\phi I_\uparrow + eA_\phi)\right) = 0\,, \tag{36}$$

$$-A\left(\sqrt{\frac{\Delta_\theta}{\rho^2}}\partial_\theta I_\uparrow + \mathrm{i}\sqrt{\frac{\rho^2\Xi^2}{\sin^2\theta\left[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta\right]}}(\partial_\phi I_\uparrow + eA_\phi)\right) = 0\,. \tag{37}$$

Our interest is still in the first two. Considering the properties of the Kerr–Newman anti-de Sitter space-time, we carry out a separation of variables as (26). Inserting the action into (34) and (35) yields

$$\left(\frac{\mathrm{i}A}{\sqrt{F(r)}}(\omega - j\Omega - eA_t) - B\sqrt{G(r)}\partial_r W(r)\right) + mA = 0\,, \tag{38}$$

$$-\left(\frac{\mathrm{i}B}{\sqrt{F(r)}}(\omega - j\Omega - eA_t) + A\sqrt{G(r)}\partial_r W(r)\right) + mB = 0\,. \tag{39}$$

In the case of $m \neq 0$, solving $W(r)$ yields

$$\begin{aligned} W_\pm(r) &= \pm\int\sqrt{\frac{(\omega - j\Omega - eA_t)^2 + m^2F(r)}{F(r)G(r)}}\,\mathrm{d}r \\ &= \pm\mathrm{i}\pi\frac{\omega - j\Omega_+ - eA_+}{\sqrt{F'(r_+)G'(r_+)}} \\ &= \pm\mathrm{i}\pi\frac{(r_+^2+a^2)(\omega - j\Omega_+ - eA_+)}{r_+ + r_+(3r_+^2+a^2)l^{-2} - (a^2+Q^2)r_+^{-1}}\,, \end{aligned} \tag{40}$$

where $+/-$ correspond to the outgoing/ingoing solutions, and $\Omega_+ = \frac{a\Xi}{r_+^2+a^2}$ and $A_+ = \frac{Qr_+}{r_+^2+a^2}$ are the angular velocity and electric potential at the event horizon, respectively. Thus the tunneling probability of the charged fermion is

$$\begin{aligned} \Gamma &= \frac{\exp(-2\,\mathrm{Im}\,I_+)}{\exp(-2\,\mathrm{Im}\,I_-)} \\ &= \exp\left(-4\pi\frac{(r_+^2+a^2)(\omega - j\Omega - eA_+)}{r_+ + r_+(3r_+^2+a^2)l^{-2} - (a^2+Q^2)r_+^{-1}}\right). \end{aligned} \tag{41}$$

So the Hawking temperature can be gotten as follows:

$$T = \frac{r_+ + r_+(3r_+^2+a^2)l^{-2} - (a^2+Q^2)r_+^{-1}}{4\pi(r_+^2+a^2)}\,, \tag{42}$$

which is in full accordance with that obtained by other methods [35] and is not related to the kind of particles. It should be emphasized that we assume that the angular momentum of the black hole does not change, considering the number of the outgoing particles with spin up to be statistically equal to that of spin down case in this paper. For the spin down case, we can adopt the same process and get the same result.

## 5 Discussion and conclusion

In this paper, extending Kerner and Mann's work to the general case, we have viewed the Hawking radiation of the Reissner–Nordström black hole, the Kerr black hole and Kerr–Newman black hole in anti-de Sitter spaces. The Hawking temperatures were recovered by fermion tunneling and are not related to the kind of particles. For the rotating black holes, after performing the dragging coordinate transformation, we find that the metrics have a similar form as that of the spherically symmetric black hole. This is convenient for us in choosing the $\gamma^\mu$ matrices and viewing the Hawking radiation of fermions; thus, we gave similar $\gamma^\mu$ matrices in this paper. In the treatment, the background space-times were regarded as fixed; therefore, the derived Hawking radiation spectra were only the leading term. When the back reaction effect and self-gravitational interaction are taken into account, the correction term of the radiation spectrum should be presented. Anyhow, we have extended Kerner and Mann's work to the general case and viewed the Hawking radiation of the black holes in anti-de Sitter spaces by the fermion tunneling method.

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Note added: after we have finished this work, the papers [37–39] appeared, which study the Hawking radiation of fermions from the Kerr black hole, the Kerr–Newman black hole and dynamical horizons.

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